An Integrated Modeling, Simulation and Analysis Environment for Coupled Aircraft Subsystems to Facilitate Control Synthesis and Validation

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This paper presents the development of an integrated modeling, simulation and analysis environment for coupled aircraft subsystems. The components of aircraft thermal and electrical power systems are modeled in Mathematica® and then integrated in MATLAB/Simulink®. Since the dynamics of such complex systems involve differential algebraic equations, a novel approach using embedded solvers was developed to achieve an accelerated simulation. The developed modeling and simulation environment is amenable to optimal control synthesis due its fast solvers and facilitates rapid validation of designed controllers. The developed tools are extensible to other aircraft components and subsystems as wells as suitable for modelling, simulation and analysis of other complex systems.

Nomenclature

c	=	heat capacity
f, g, h	=	generic functions
k	=	constant
l, m, n	=	space dimensions
m	=	mass flow rate
p	=	number of pole pairs
q	=	volumetric flow rate
t	=	continuous time index
u	=	inputs
v	=	volume
x	=	state vector
y	=	measurements
A	=	area
B	=	damping
D	=	constant
E_f	=	field voltage
I	=	current
J	=	inertia
L	=	inductance
P	=	real power

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P_m	=	mechanical power
P_a, P_b	=	port pressure
P_p	=	pressure
Q	=	reactive power
Q_d	=	conductive heat flow
Q_v	=	convective heat flow
Q_T	=	total heat flow
R	=	resistance
V	=	voltage
T	=	temperature
T_a, T_b	=	port temperature
T_e	=	electrical torque
T_m	=	mechanical torque
X	=	reactance
Z	=	impedance
δ	=	rotor angle
η	=	efficiency
ϕ	=	magnetic flux
ρ	=	density
θ	=	voltage angle
ω	=	angular velocity

I. Introduction

MODELING and simulation technologies perform an important role in control system synthesis, and validation and verification of the designed controllers rapidly. This is especially important for aircraft systems where multiple interacting subsystems need to be analyzed and controlled simultaneously. There are several important aspects to consider for modeling and simulation of aircraft subsystems. For example, the underlying continuous dynamics of each component that constitute the subsystems are usually well-understood; however, this is not sufficient to express possibly nonlinear feedback and feed-forward relationships between components. The problem is further complicated with existence of discrete transitions such as discrete action of protection devices, switches and lower level control systems, and external disturbances such as human inputs. Another complicating issue is the structure of the mathematical equations that describe the model behavior. For example, electric power system dynamics are usually described by ordinary differential equations (ODE), however, power network imposes algebraic constraints on the system, i.e. how the components are connected to each other. Therefore, the full mathematical description of the system behavior is in the form of differential algebraic equations (DAE).[1-3]

Our motivation is to develop a comprehensive aircraft energy management system that is based on a mathematical framework with a variable cost function that can be optimized, while addressing individual mathematical relationships to each subsystem. The control design should also take into account the fact that the subsystems may be operated close to stability limits and that many subsystems have local protection mechanisms which interfere with the control. For such control problems, modeling and simulation methods can be employed very effectively for optimization studies and control synthesis.[4-6] The drawback here has been traditionally associated with the fundamental trade-off between the computational speed and the accuracy of the computations. If a high-fidelity model is employed for Monte-Carlo type optimization, the computation time drastically increases for the control synthesis.[7] Whereas, the low-order models can accomplish optimization goals at a much faster rate. However, the resulting controller may not be adequate for implementation.[8] To this end, an accurate model of the thermal and electrical subsystems is required that is also amenable for control synthesis and optimization.

This paper presents a set of modeling, simulation and management design tools to facilitate the design and operation of coupled aircraft subsystems. A switched, differential-algebraic description of the system is used to capture the wide ranging time scales that are necessary to understand the full operational envelope of the subsystems. These models provide insight into the inherent dynamic constraints of the aircraft subsystems which have both discrete and nonlinear behavior. This formulation provides a framework for the synthesis of optimal control algorithms. A novel methodology is used to simulate the switched, nonlinear behavior of the system that enables resourceful computation of system variables and outputs. This simulation capability allows the rapid prototyping of associated control algorithms. These advances are demonstrated on a notional twin-engine aircraft system that includes a thermal system, and an electric power system with necessary hydraulic and mechanical interfaces.

In this paper, we also describe the software environment that leverages the strengths of widely available scientific software such as Mathematica and MATLAB/Simulink. The computational tools for model assembly are provided in symbolic environment of Mathematica which also has tools for nonlinear control design. The symbolic models are integrated in the simulation environment of MATLAB/Simulink. The novelty of this approach is the ability to create custom solvers in the symbolic environment that enables very fast simulation of the high-fidelity models. Therefore, these simulation models can be used for optimal control synthesis, and open-loop and closed-loop simulations for controller validation and verification.

This paper is organized in five sections including the present one. Section II elaborates the approach for modeling and accelerated simulation of aircraft subsystems. Section III describes the various component models and simulation development. In section IV the simulations results for the coupled electrical and thermal system with mechanical and hydraulic interfaces are presented. The paper is summarized and concluded in section V. The component model equations are presented in the Appendix.

II. Modeling and Simulation Approach

The ability to run very fast simulations of coupled subsystems is essential to designing and implementing intelligent, high performance aircraft management systems. However, fast simulation is impeded by the structure of subsystem models which are in the general form of differential-algebraic equations.[9] For example, MATLAB/Simulink, one of the most important model building and simulation tools for control system designers, does not contain any reliable computational tools for DAEs. Our approach introduces a new method of addressing modeling and simulation of coupled subsystems within Simulink.

This approach views the system in terms of a "network" which embodies the critical algebraic equations to which other components of various types and their controllers are attached to. The network algebraic equations are implemented as a discrete time component that implements one or more iteration of Newton's method. The assembly of the network model is accomplished symbolically resulting in an optimized C-code program that compiles as a Simulink S-function.[10]

Consider a system that is typically represented by DAEs of the semi-implicit form

$$\begin{aligned} \dot{x} &= f\left(x,u\right) \\ y &= g\left(x,u\right) \\ 0 &= h\left(x,u\right) \end{aligned} \tag{1}$$

where $t \in \mathbb{R}^n$, $t \geq t_0$ is the time index, $x \in \mathbb{R}^n$ is the n-dimensional state of the system, $u \in \mathbb{R}^m$ is the m-dimensional input of the system and $y \in \mathbb{R}^l$ is the l-dimensional output of the system. $f\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $g\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^l$ are continuous vector fields that defines the evolution of the system dynamics and $h\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ defines the algebraic constraints.

As noted earlier, reliable and efficient methods for solving general DAEs are currently unavailable in simulation tools like Simulink. In prior work, we implemented a variant of the 4^{th} and 5^{th} order Runge-Kutta-Fehlberg method (this is one of MATLAB's standard tools for solving ordinary differential equations) to solve the semi-implicit DAE models. At each time step, we used a standard Newton-Raphson procedure to solve the algebraic equations for x.[4] This is feasible so long as the trajectory does not pass through a 'non-causal' point. Non-causal points and the related 'singularity-induced' bifurcation has been studied earlier.[11]

In the present approach, this is extended so that it can be used in the Simulink environment. This allows engineers to use Simulink's graphical interface to build and share models in a modular framework and to exploit Simulink's computational capabilities and its various toolboxes. While toolboxes exist for physical modeling within Simulink (e.g., Simscape, SimMechanics, SimPowerSystems, etc.), they solve the DAE computational issue by including enough parasitic dynamics in the models to break all of the many 'algebraic loops' thus producing a set of 'stiff' differential equations instead of a system of DAEs. The resulting simulations are documented to take a long time to resolve.[12,13]

For example, an aircraft electric power subsystem is a collection of components including generators, motors, storage devices, static loads and others connected together by a network of transmission lines. Building a mathematical model of the system can be facilitated by a graphical interface in which models of individual components and the network can be connected together in the desired configuration. Simulink provides such an interface where the system model can be assembled, modified and simulated as part of a design or analysis process. The approach taken in this paper is to isolate the network thereby isolating all of the algebraic loops in a single block that implements a number of Newton iteration as a discrete time dynamical element. The simulation can be organized so that one or more Newton iterations are performed during each continuous time step. The same architecture can used for other aircraft subsystems such as the thermal subsystem with its thermal masses, convective and conductive heat transfer mechanisms. The subsystems are then interconnected through the Simulink's graphical interface for a complete system description. The result is a very fast simulation due to eliminating the artificial parasitic dynamics.

III. Component Models and Simulation Development

A working model was developed for a notional aircraft thermal and electric power system to allow for design, analysis and validation of optimal controllers developed for this system. Notional two-engine aircraft model can accommodate power generation plant, integrated power unit for electrical power, cooling and thermal system, electric and thermal loads as well as associated component models for actuation and sensing.

The general structure of the network is defined in Fig. 1. It is viewed as a multi-port block. Each port (or bus) interacts with the external world through four variables P, Q, V, θ. Two of these are inputs and two are outputs. The most common input pairs are P, Q, P, V, and V, θ.

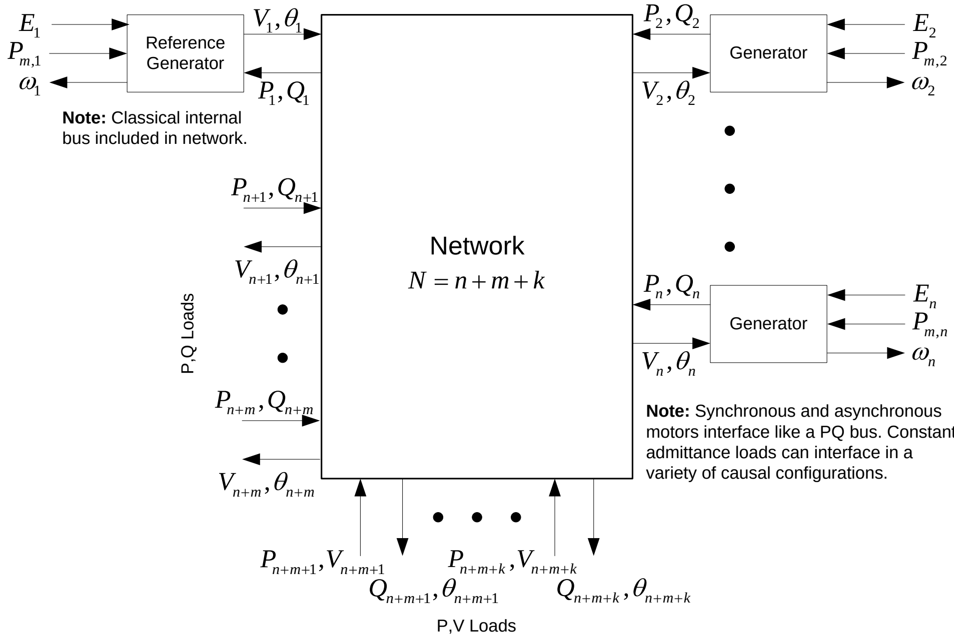


Figure 1. The network as a multi-port block.

Note that because of the translational symmetry in the network equations at least one bus must have θ as an input variable. This would most likely be a reference generator, corresponding to a V, θ bus. The connectivity matrix for a 4-bus aircraft power distribution network is given in the Appendix.

There are various models of the power system components with different complexities.[14] The simplest synchronous generator model represents the electrical side as a voltage behind the transient reactance. The equations are given in the Appendix. There are two ways to deal with this model. The conventional approach is to consider it as a distinct, self-contained block with inputs field voltage, E_f, terminal bus voltage magnitude and angle, V, θ, and mechanical power, P_m. The outputs are rotor angle, δ, rotor angular velocity, ω, and terminal bus real and reactive power injections, P_2, Q_2. This is called a PQ model. The second approach is to incorporate the reactance in the network. In this case, the generator block consists only of the mechanical equations. The inputs are field voltage, E_f, terminal bus real power, P_1, and mechanical power, P_m. The outputs are rotor angle, δ, rotor angular velocity, ω, and internal bus voltage magnitude and angle, V_1, θ_1. This is called a reference model. The Simulink model is given in Fig. 2.

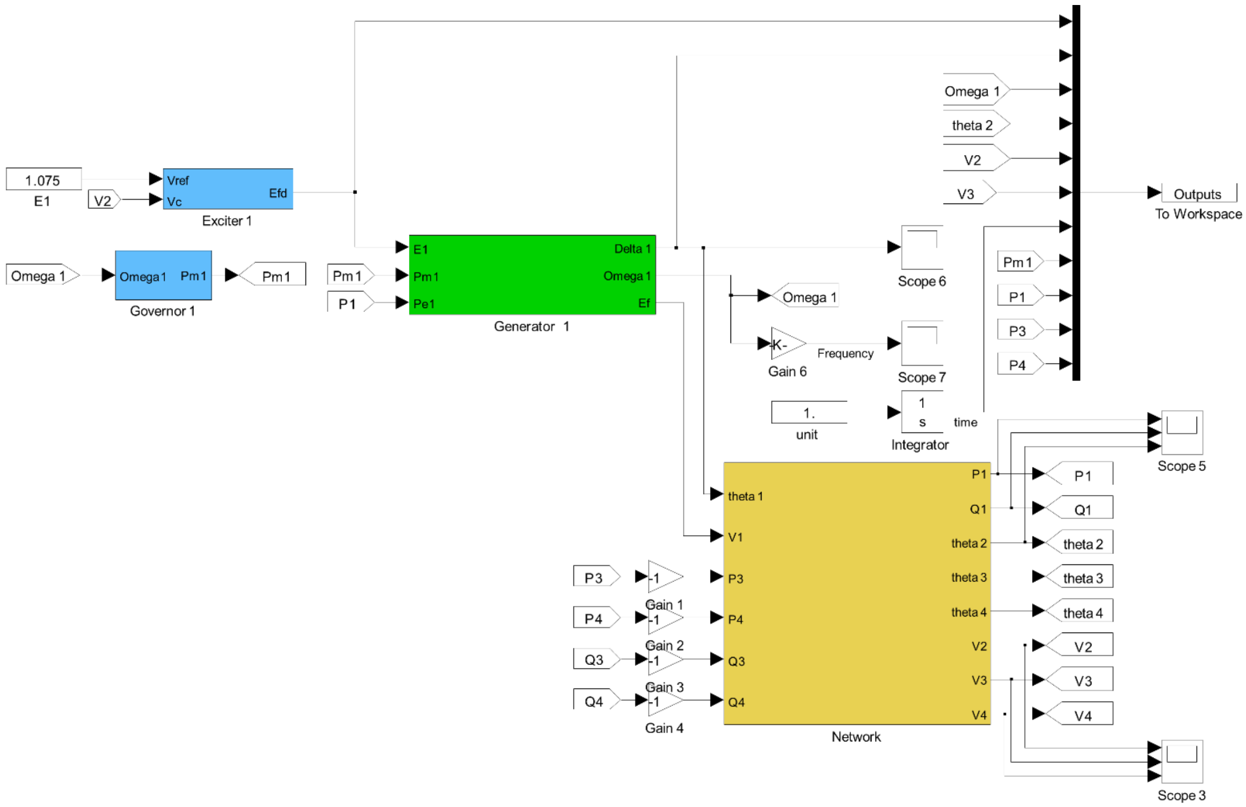


Figure 2. Generator and network implementation.

Each electric actuator is modeled as a permanent magnet synchronous motor (PMSM). The electrical machine state equations are modeled in what is known as the d-q reference frame. The equations are given in the Appendix. The inputs are the mechanical torque T_m and voltage V. The outputs are current, I, electrical torque, T_e, and rotor angular speed, ω_r. The Simulink model is given in Fig. 3. Two of these models with appropriate loading profile are implemented in simulation.

The simplest electric load type is a linear, constant admittance (or impedance) load. The impedance is typically written as $Z = R + iX$. Therefore, the power consumed can be written in terms of impedance. The implementation used here has inputs voltage, V, resistance, R, and reactance, X. The outputs are the real and reactive power, P and Q, respectively.

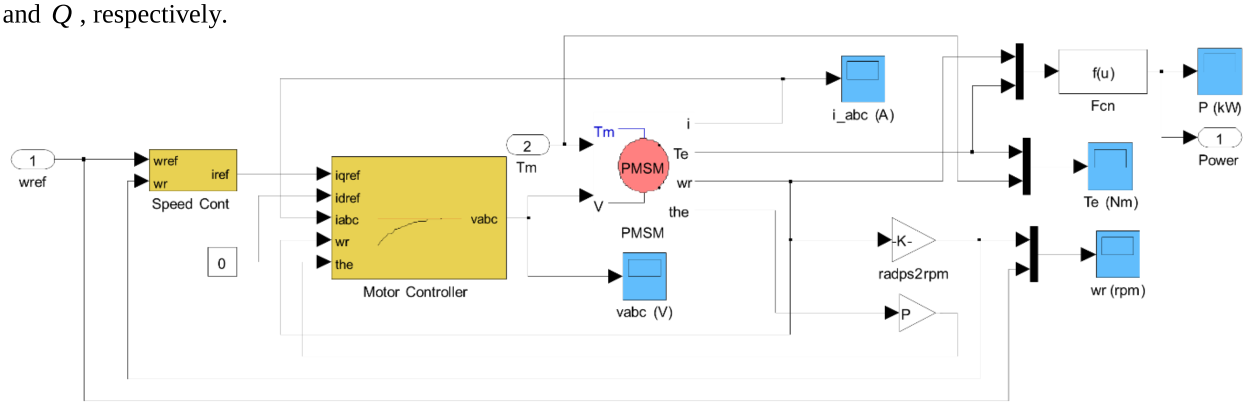


Figure 3. PMSM implementation.

Two separate hydraulic pumps are assumed to circulate the fuel and oil through the heat exchanger. Fixed displacement hydraulic pump equations are given in the Appendix. The inputs are, mechanical torque, T_m, angular velocity, ω, and input pressure, P_a. The outputs are volumetric flow rate, q, and output pressure, P_b. The Simulink model is shown in Fig. 4.

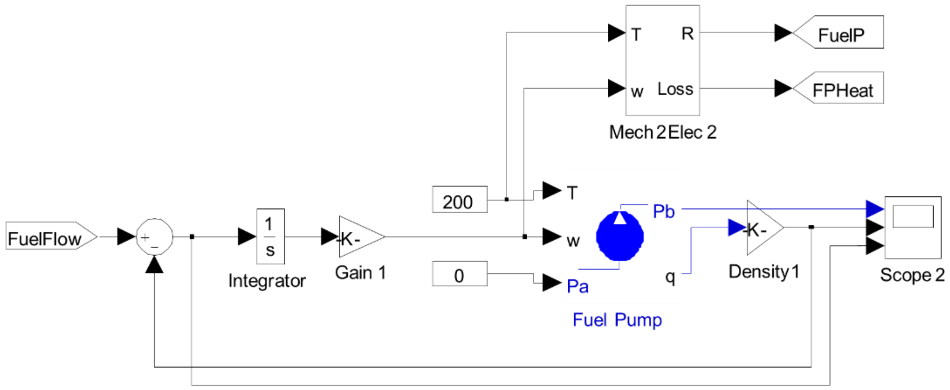


Figure 4. Fixed displacement pump implementation.

The thermal subsystem implemented for the simulation can be seen in Fig. 5. The conductive and convective heat transfer functions and thermal mass for oil and fuel are modeled. The short settling time is a function of the efficiency of the convective and conductive heat transfer. The model equations are given in the Appendix. Thermal mass has mass flow rate, m, and total heat flow, Q_T as inputs and temperature, T, as output. Heat transfer functions have two boundary temperatures, T_a, T_b, as input and heat flow, Q, as output.

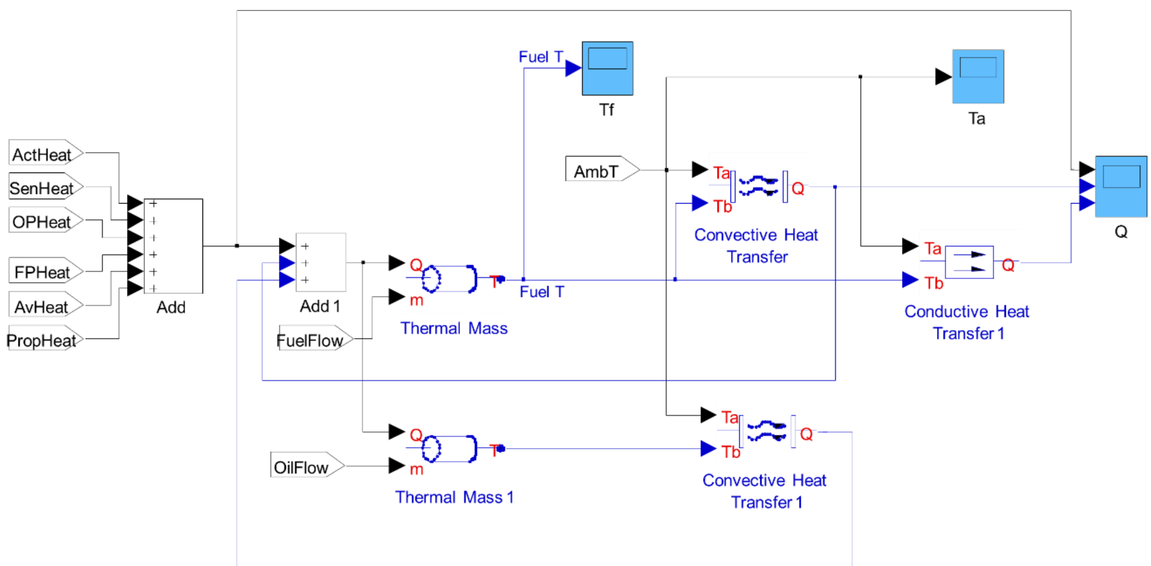
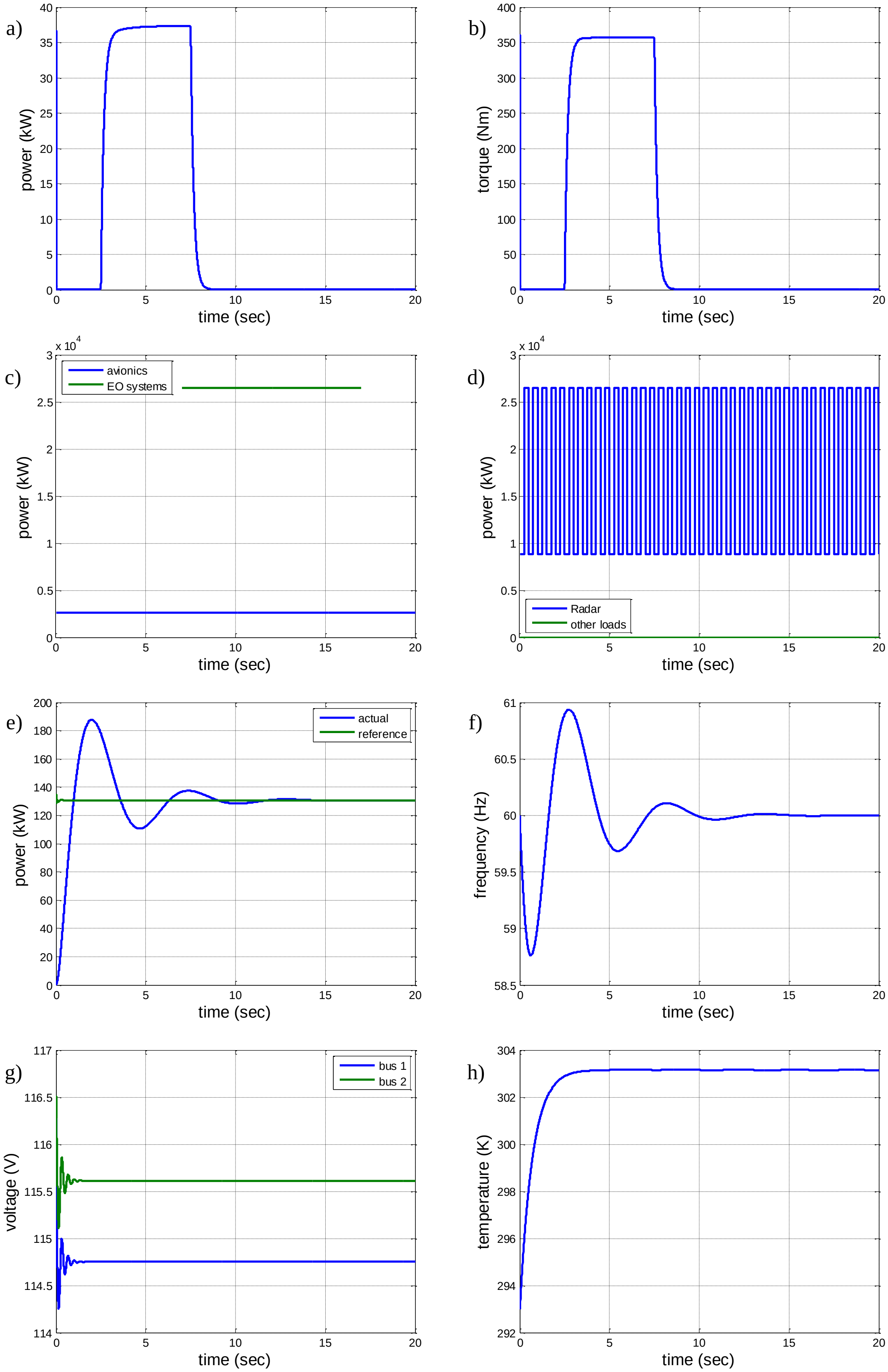


Figure 5. Thermal system implementation

IV. Simulation Results

Once the component model equations are assembled in Mathematica, an optimized C-code is generated for each model and they are exported to Simulink using the MEX interface. The simulation models then integrated using the graphical interface of Simulink as described in Section III. The complete system is simulated for 20 seconds to obtain first level validation of the models. The simulation outputs are given in Fig. 6.



a)
power (kW)
time (sec)
b)
torque (Nm)
time (sec)
c)
x 10^4
avionics
EO systems
power (kW)
time (sec)
d)
x 10^4
Radar
other loads
power (kW)
time (sec)
e)
actual
reference
power (kW)
time (sec)
f)
frequency (Hz)
time (sec)
g)
bus 1
bus 2
voltage (V)
time (sec)
h)
temperature (K)
time (sec)

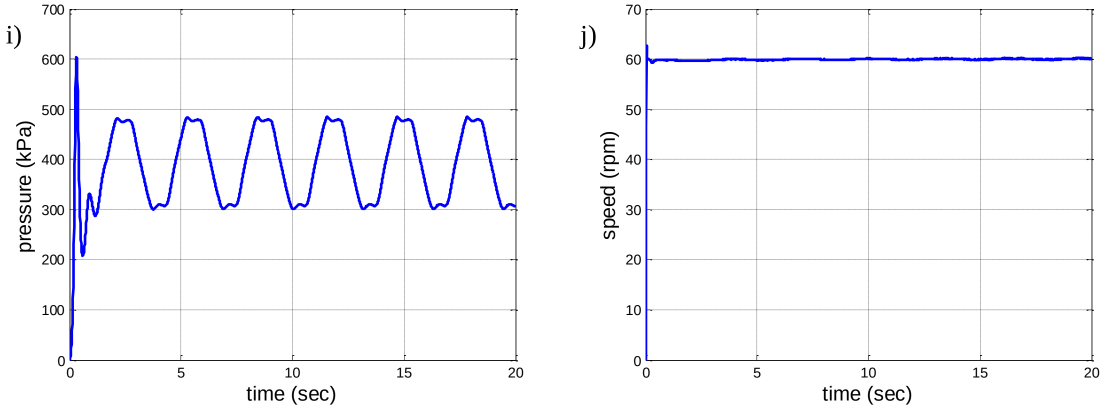


Figure 6. 20-second simulation results for the aircraft system components. a) PMSM power output (kw). b) PMSM torque output (N.m). c) Avionics and EO systems power demand (kW). d) Radar power demand (kW). e) Generator power output (kW). f) Generator frequency (Hz). g) Network generator bus voltages (V). h) Fuel temperature (K). i) Fuel pump pressure (kPa). j) Fuel pump angular speed (rpm)

Note that these models are representative and are designed to capture only the essential dynamics of the heat transfer mechanism, electric loads and hydraulic dynamics. Greater detail can be added for verification purposes as needed. The elegance of our approach is that it is only needed to modify the component models and not the overall simulation. Also, it is traditionally accepted that control strategies are to be develop with simpler models and be tested against higher fidelity simulations to get greater confidence in case there is a model mismatch i.e., when models do not capture physical reality exactly.

The computational speed of the simulation was tested on three laptop computers. Each laptop has a different hardware configuration in terms of processor speed and random access memory, but all have Windows 7 operating system. The simulation environment for each case was MATLAB R2011b. The simulations were executed in 'normal' speed of Simulink and MATLAB's ode45 solver was used. The results were compiled for 10 simulations. The simulation speed was calculated to be between 3.8× real-time to 6.5× real-time and on the average of 4.4× real-time. Real-time is defined as the elapsed processor time for each unit time (e.g. seconds) of simulation. Although a variable-step solver is used, the total number of simulation steps in each case was the same. The wide discrepancy in the results is due to the different hardware configuration as well as the other active processes at the time of simulations.

V. Conclusion

This paper presented a framework and computational tools relevant to the modeling and simulation of coupled aircraft systems. The computational tools for modeling and simulation were implemented using widely available scientific software which is useful for validation and verification studies. The important contribution is the ability to structure the simulation so that standard Simulink solvers can be utilized to predict the nonlinear, switched dynamics of coupled systems in reasonable timeframe on personal computers. These tools are expected to aid rapid insertion of advanced optimization and control technologies into next generation more-electric aircraft frames.

Appendix

This Appendix provides the mathematical description of component models used in the modeling and simulation environment of this paper.

The connectivity matrix for the 4-bus network is given in terms of reactance X as:

$$\begin{pmatrix} -X & X & 0 & 0 \\ X & -5X & 2X & 2X \\ 0 & 2X & -2X & 0 \\ 0 & 2X & 0 & -2X \end{pmatrix} \tag{2}$$

The equations for synchronous generator are:

$$
\begin{aligned}
&\frac{d\theta}{dt} = \omega - \omega_s \\
&\frac{d\omega}{dt} = \frac{\omega_s}{2H}\left(P_m - \left[E_f V \sin\left(\delta - \theta\right)\right]/X - D_m\left(\omega - \omega_s\right)\right) \\
&P_1 = \left[E_f V \sin\left(\delta - \theta\right)\right]/X \\
&Q_1 = -\frac{E_f^2}{X} + \left[E_f V \cos\left(\delta - \theta\right)\right]/X \\
&P_2 = \left[E_f V \sin\left(\delta - \theta\right)\right]/X \\
&Q_1 = -\frac{V^2}{X} + \left[E_f V \cos\left(\delta - \theta\right)\right]/X
\end{aligned}
\tag{3}
$$

The equations for PMSM are:

$$
\begin{aligned}
&\frac{dI_d}{dt} = \frac{1}{L_d}\left(V_d - RI_d + \omega L_q I_q\right) \\
&\frac{dI_q}{dt} = \frac{1}{L_q}\left(V_q - RI_q - \omega L_d I_d - \omega\phi\right) \\
&\frac{d\omega_r}{dt} = \frac{1}{J}\left(T_e - T_m - B\omega_r\right) \\
&\omega = p\omega_r \\
&T_e = \frac{3}{2}p\left(I_q\phi - \left(L_q - L_d\right)I_q I_d\right)
\end{aligned}
\tag{4}
$$

The equations for static electric load are:

$$
\begin{aligned}
&P = \frac{RV^2}{R^2 + X^2} \\
&Q = \frac{XV^2}{R^2 + X^2}
\end{aligned}
\tag{5}
$$

The equations for hydraulic pump are:

$$
\begin{aligned}
&k_{HP} = \frac{1}{\bar{P}_p}\left(D\bar{\omega}\left(1 - \eta_v\right)\bar{v}\rho\right) \\
&k_l = \frac{k_{HP}}{v\rho} \\
&P_p = \frac{1}{D}T_m\eta_m \\
&q = D\omega - k_l P_p \\
&P_b = P_a + P_p
\end{aligned}
\tag{6}
$$

The equations for heat transfer are:

$$\frac{dT}{dt} = \frac{Q_T}{cm}$$
$$Q_v = k_v A\left(T_a - T_b\right) \qquad (7)$$
$$Q_d = \frac{1}{D} k_d A\left(T_a - T_b\right)$$

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